# Effect of Ellipticity on Dominant-Mode Axial Ratio in Nominally Circular Waveguides

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*Summary*—An investigation is made of the effect of ellipticity on the dominant-mode axial ratio (AR) in nominally circular waveguides. Equations for calculating the AR are derived for the case where the difference between the major and minor axes of the guide cross section is small and the waveguide is not too long. Values of AR obtained by calculations are compared with measured values, and a method for improving the AR performance of a waveguide run is demonstrated.

## Introduction

AXIAL ratio (AR) is defined as the ratio of the major axis to the minor axis of the polarization ellipse.[1] For a dominant mode ($TE_{11}$) in a circular or elliptical waveguide, the AR at any point $z$ along the guide is the ratio of the maximum to the minimum rms component of the electric field at the center of the guide at $z$, when it is terminated in its characteristic impedance.

The AR performance of an elliptical waveguide is important when the guide is used for transmitting two orthogonal dominant-mode signals. If the waveguide were perfectly circular, the AR would always be infinite and there would be no interaction between the two signals. Most commercially produced, nominally circular waveguides, however, are actually out of round. For an arbitrary polarization of the incident signals, the AR is then in general finite and will introduce crosstalk. The amount of crosstalk increases as the AR decreases.

This paper discusses the effect of ellipticity on the dominant-mode AR in nominally circular waveguides. Equations are derived for calculating the AR when the difference between the major and minor axes of the guide cross section is small and the waveguide is not too long. Calculated values of AR are checked by measurements, and a method for improving the AR performance of a waveguide run is demonstrated.

## Discussion

At the input of an elliptical waveguide the dominant-mode electric field AR the center of the guide can be represented as shown in Fig. 1, where

$$E_{\text{inc}} = A \cos \omega t \tag{1}$$

is the maximum component of the electric field incident on the waveguide. $E_{\text{inc}}$ can be resolved into two other components

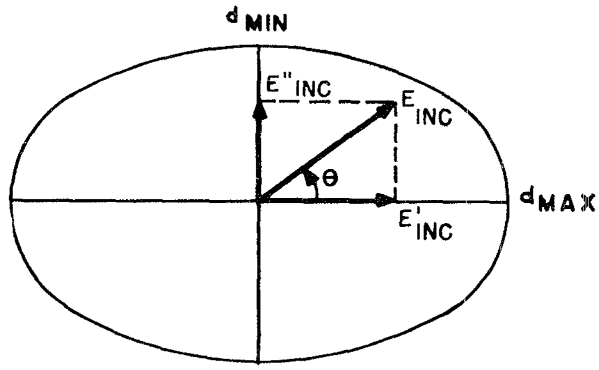


Fig. 1—Cross section of an elliptical waveguide, showing the incident electric field.

$$E_{\text{inc}}' = A \cos \theta \cos \omega t \tag{2}$$

$$E_{\text{inc}}'' = A \sin \theta \cos \omega t, \tag{3}$$

where $E_{\text{inc}}'$ is parallel to the major axis ($d_{\max}$) of the guide cross section, $E_{\text{inc}}''$ is parallel to the minor axis ($d_{\min}$), and $\theta$ is the angle between $E_{\text{inc}}$ and $d_{\max}$. $E_{\text{inc}}'$ is called an odd dominant mode and $E_{\text{inc}}''$ an even dominant mode.[2]

Let us assume that these two modes are traveling down the waveguide, which is terminated in its characteristic impedance. At a distance $z$ from the input of the guide we have

$$E' = A \cos \theta \cos (\omega t - \beta' z) \tag{4}$$

$$E'' = A \sin \theta \cos (\omega t - \beta'' z), \tag{5}$$

where $\beta'$ is the phase constant of the odd mode and $\beta''$ the phase constant of the even mode. This gives the magnitude of the resultant field at $z$

$$|E| = A[\cos^2 \theta \cos^2 (\omega t - \beta' z) + \sin^2 \theta \cos^2(\omega t - \beta'' z)]^{1/2}. \tag{6}$$

A physical picture of behavior of $E$ can be obtained as in Fig. 2 (next page). Here $E'$ and $E''$ are represented as phasors rotating counterclockwise with angular velocity $\omega$ and out of phase by an angle $(\beta'' - \beta')z$. It is seen that as $t$ varies, $E$ changes in both magnitude and its orientation relative to $d_{\max}$ or $d_{\min}$.

Now AR is defined by the equation

$$AR = \frac{|E|_{\max}}{|E|_{\min}}. \tag{7}$$

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[1] "IRE standards on antennas and waveguides: definitions of terms, 1953," vol. 41, pp. 1721–1728; December, 1953.

[2] L. J. Chu, "Electromagnetic waves in elliptic hollow pipes of metal," *Jour. Appl. Phys.*, vol. 9, pp. 583–591; September, 1938.

The maximum and minimum values of $|E|$ are obtained by setting

$$\frac{d|E|}{dt} = 0. \tag{8}$$

This gives

$$\sin 2(\omega t - \beta' z) + \tan^2\theta \sin 2(\omega t - \beta'' z) = 0 \tag{9}$$

or

$$\sin(2\omega t - \beta'' z)[\cos 2(\beta'' - \beta')z + \tan^2\theta] + \cos 2(\omega t - \beta'' z)\sin 2(\beta'' - \beta')z = 0. \tag{10}$$

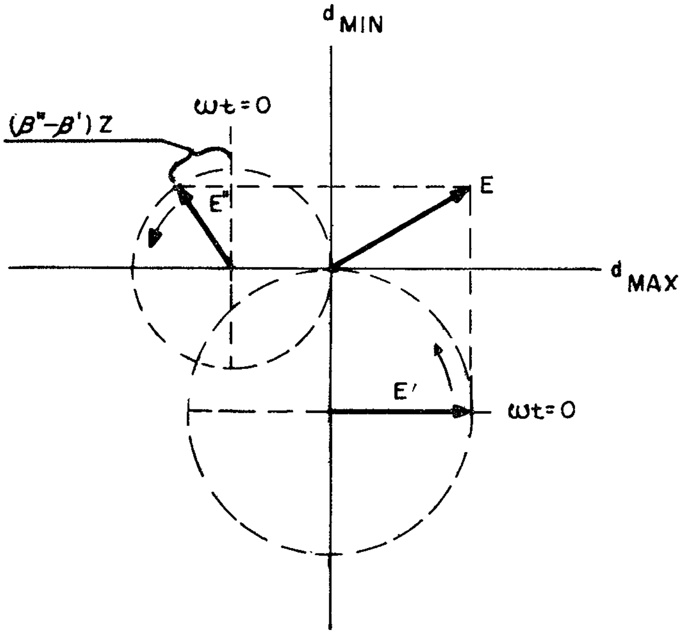


Fig. 2—Representation of the electric field at a point $z$ along the waveguide.

Solving for $\omega t$, we have

$$\omega t = \beta'' z - \frac{1}{2}\tan^{-1}\left[\frac{\sin 2(\beta'' - \beta')z}{\cos 2(\beta'' - \beta')z + \tan^2\theta}\right]. \tag{11}$$

If we now assume

$$2(\beta'' - \beta')z \ll 1, \tag{12}$$

we can set

$$\sin 2(\beta'' - \beta')z \cong 2(\beta'' - \beta')z \tag{13}$$

$$\cos 2(\beta'' - \beta')z \cong 1. \tag{14}$$

Then from (11)

$$\omega t \cong \beta'' z - (\beta'' - \beta')z\cos^2\theta + \frac{n\pi}{2} \quad (n = \text{any integer}), \tag{15}$$

giving

$$\omega t - \beta' z \cong (\beta'' - \beta')z\sin^2\theta + \frac{n\pi}{2} \tag{16}$$

$$\omega t - \beta'' z \cong -(\beta'' - \beta')z\cos^2\theta + \frac{n\pi}{2}\cdot \tag{17}$$

Substituting from (16) and (17) in (6), $|E|_{\max}$ is obtained when $n$ is even and $|E|_{\min}$ is obtained when $n$ is odd. Then by using (7),

$$AR = \frac{1}{\frac{z}{2}|\beta'' - \beta'|\,|\sin 2\theta|}\cdot \tag{18}$$

The AR is minimum (and the crosstalk maximum) when $\theta = 45$ degrees, that is when the magnitudes of $E'$ and $E''$ are equal. We can therefore write

$$AR = \frac{AR_{\min}}{|\sin 2\theta|}, \tag{19}$$

where

$$AR_{\min} = \frac{1}{\frac{z}{2}|\beta'' - \beta'|}\cdot \tag{20}$$

Expressing $\beta'$ and $\beta''$ in terms of the guide wavelengths of the odd and even modes

$$\lambda_g' = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c'}\right)^2}} \tag{21}$$

$$\lambda_g'' = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c''}\right)^2}}, \tag{22}$$

(20) can be written

$$AR_{\min} = \frac{1}{\frac{\pi z}{\lambda}\left|\sqrt{1 - \left(\frac{\lambda}{\lambda_c''}\right)^2} - \sqrt{1 - \left(\frac{\lambda}{\lambda_c'}\right)^2}\right|}\cdot \tag{23}$$

Here $\lambda$ is the wavelength in free space, $\lambda_c'$ the cut-off wavelength for the odd mode, and $\lambda_c''$ the cut-off wavelength for the even mode. $\lambda_c'$ and $\lambda_c''$ are given by the equations[3]

$$\lambda_c' = \frac{\pi d_{\max}}{k'} \tag{24}$$

$$\lambda_c'' = \frac{\pi d_{\max}}{k''}, \tag{25}$$

where

$k'$ and $k''$ are the roots[4]

$$k' = k(1 + .48e^2 + .035e^4 + \cdots) \tag{26}$$

$$k'' = k(1 + .02e^2 + .0034e^4 + \cdots). \tag{27}$$

$k = 1.841$ is the first root of the Bessel function $J_1'(k) = 0$ and

[3] Unpublished work by A. P. King of Bell Telephone Laboratories, Inc.

[4] Roots were derived by Marion C. Gray of Bell Telephone Laboratories, Inc.

$$e = \frac{\sqrt{d_{\max}^2 - d_{\min}^2}}{d_{\max}} \tag{28}$$

is the eccentricity of the guide.

Assuming

$$d_{\max} - d_{\min} \ll d_{\max}, \tag{29}$$

then

$$e^2 \ll 1 \tag{30}$$

and we can set

$$\lambda_c' \cong \frac{\pi d_{\min}}{k} \tag{31}$$

$$\lambda_c'' \cong \frac{\pi d_{\max}}{k}. \tag{32}$$

Substituting for $\lambda_c'$ and $\lambda_c''$ in (23), we get

$$AR_{\min} = \frac{1}{\frac{\pi z}{\lambda}\left[\sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d_{\max}^2}} - \sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d_{\min}^2}}\right]}. \tag{33}$$

To obtain a simpler form for (33), let

$$d = \tfrac{1}{2}(d_{\max} + d_{\min}) \tag{34}$$

$$\Delta = d_{\max} - d_{\min}. \tag{35}$$

Then

$$\sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d_{\max}^2}} - \sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d_{\min}^2}}$$

$$\cong \Delta \frac{d}{dx}\sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 x^2}}\bigg|_{x=d} = \frac{\frac{\Delta}{d}\frac{\lambda^2 k^2}{\pi^2 d^2}}{\sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d^2}}}, \tag{36}$$

giving

$$AR_{\min} = \frac{\sqrt{1 - \frac{\lambda^2 k^2}{\pi^2 d^2}}}{\frac{\pi z}{\lambda}\frac{\Delta}{d}\frac{\lambda^2 k^2}{\pi^2 d^2}}. \tag{37}$$

If $\phi$ is the angle between the maximum component of the electric field at the input and the maximum component at the output of an elliptical waveguide, then[5]

$$\cot \phi \cong AR. \tag{38}$$

Where (12) holds true,

$$AR \gg 1 \tag{39}$$

and we can set

$$\phi \cong 0. \tag{40}$$

[5] A. P. King, "Dominant wave transmission characteristics of a multimode round waveguide," Proc. IRE, vol. 40, pp. 966–969; August, 1952.

Whence it can be assumed that if two sections of elliptical guide are connected in series, the maximum electric field incident on the first section has the same orientation as the maximum field incident on the second section. The total AR of the two waveguides in series is therefore given by the equation

$$AR_T = \frac{1}{\left|\frac{\sin 2\theta'}{AR_{\min}'} + \frac{\sin 2\theta''}{AR_{\min}}\right|}. \tag{41}$$

## Calculations

Using (37), the effect of ellipticity on the dominant-mode AR in a section of 2.812 id nominally circular wave-guide was calculated. The section was assumed to be 8 inches long, and $d_{\max}$ was held constant at 2.812 inches, while $d_{\min}$ was decreased in steps from 2.807 to 2.762 inches. The calculations were made at 3,950 and 6,175 megacycles, and the results are plotted as $AR_{\min}$ in db vs $d_{\max} - d_{\min}$ in inches in Fig. 3 and Fig. 4 (next page).

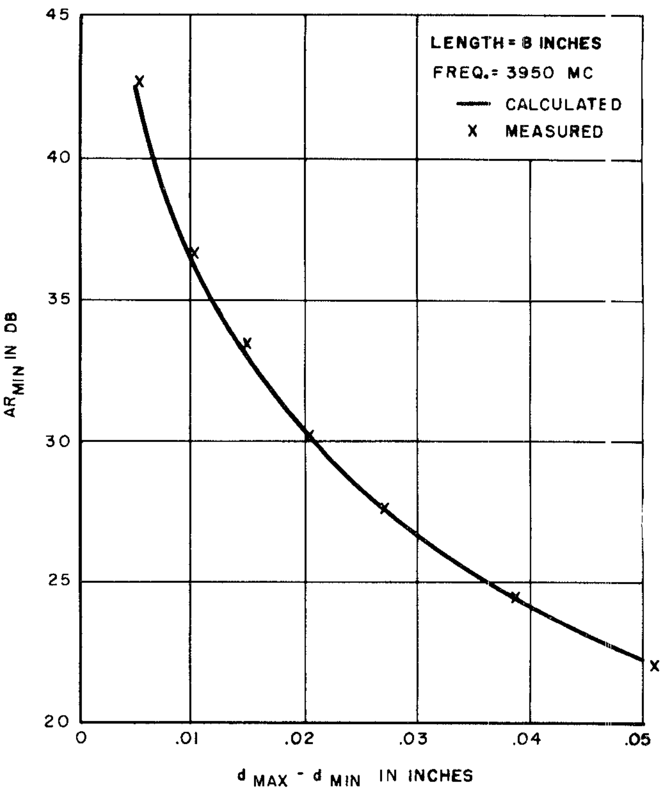


Fig. 3—Minimum axial ratio at 3950 mc vs difference between major and minor axes for an 8-inch section of 2.812 id nominally circular waveguide.

Assuming an $AR_{\min}$ of 25 db, the AR of a waveguide section when rotated in steps through 180°, was calculated from (19). The values of AR in db are plotted vs $\theta$ in degrees in Fig. 5 (next page).

The $AR_T$ of two waveguide sections in series was calculated by using (41). $AR_{\min}'$ and $AR_{\min}''$ were both 25 db. One section was held fixed at $\theta' = 45°$; the other

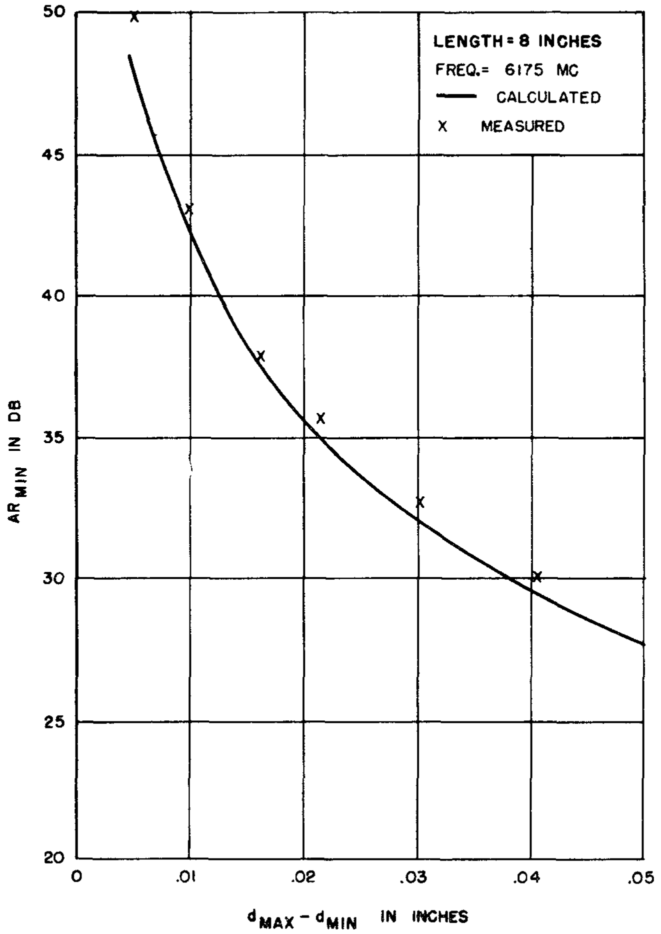


Fig. 4—Minimum axial ratio at 6175 mc vs difference between major and minor axes for an 8-inch section of 2.812 id nominally circular waveguide.

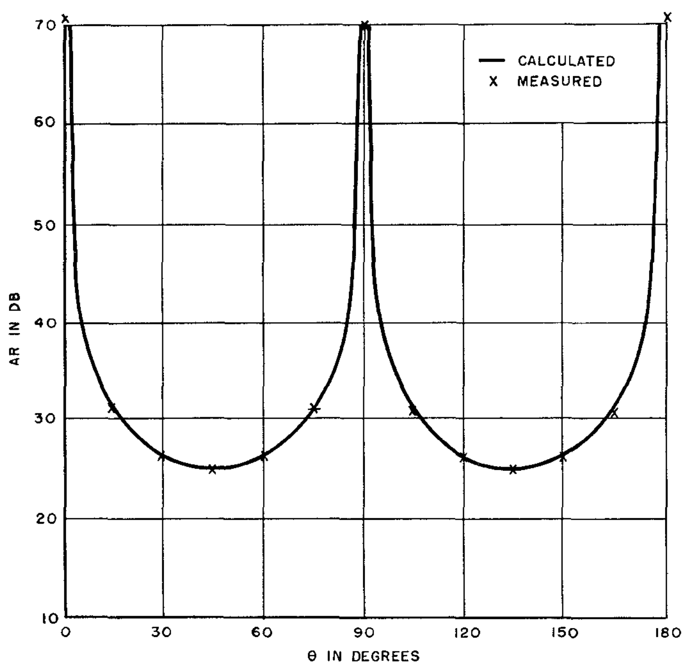


Fig. 5—Axial ratio of a waveguide section vs orientation of the section.

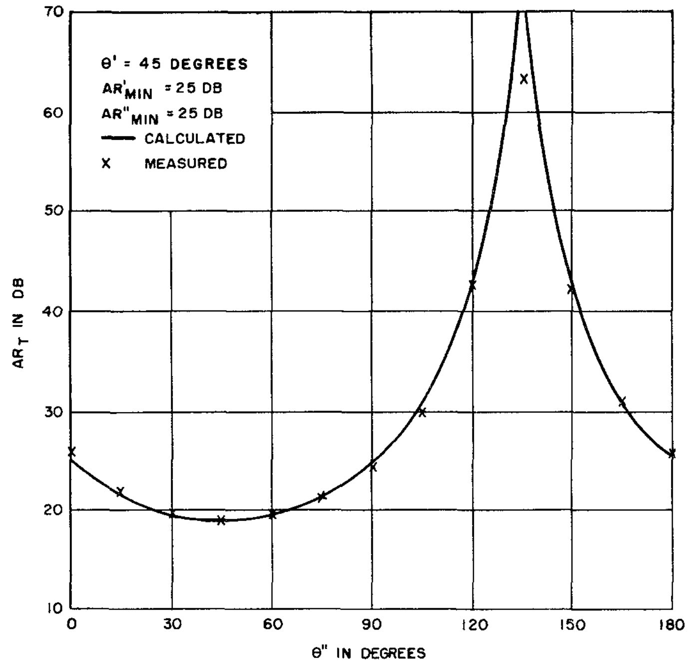


Fig. 6—Total axial ratio of two waveguide sections connected in series vs orientation of one of the sections.

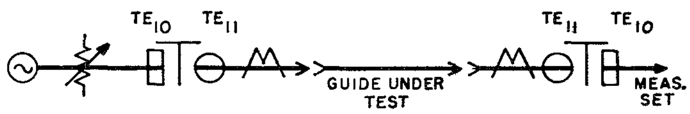


Fig. 7—Test setup for measuring axial ratio.

section was rotated in steps through 180°. Results are plotted as $AR_T$ in db vs $\theta''$ in degrees in Fig. 6 (next page).

The calculations show that the AR increases as the frequency is increased. This is due to the fact that at higher frequencies the guide is oversize for the dominant mode, thus making the mode less dependent on the guide wall. The $AR_T$ is lowest when the major axes (or minor axes) of the two waveguide sections are parallel to each other and since here $AR_{min}' = AR_{min}''$, the $AR_T$ is infinite when the major axes are perpendicular to each other.

## Measurements

The results of the calculations made above were checked experimentally by means of the test setup shown in Fig. 7. A mode filter was used between the waveguide under test and each of the two $TE_{10}^{\square} \rightleftarrows TE_{11}^{\circ}$ transducers in order to attenuate any higher order modes which might be present. Both mode filters consisted of a resistive vane (Synthane, Grade L-564) 11 inches long and with a 3-inch taper at each end as in Fig. 8 (next page). The vane was placed perpendicular to maximum electric field of dominant mode, and it attenuated by more than 40 db any component of the electric field parallel to the vane. The maximum and minimum components of the electric field at the output of the waveguide were measured by rotating together the output mode filter and transducer.

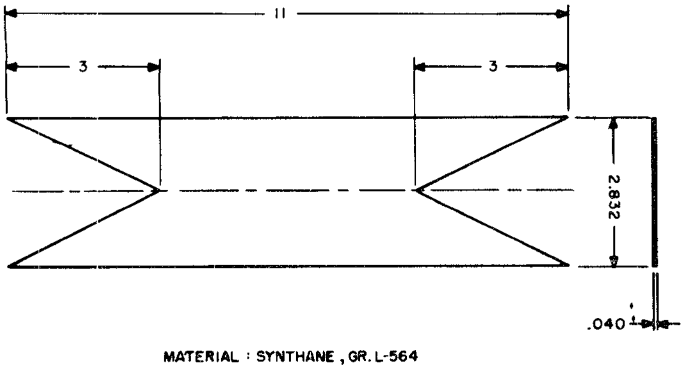


Fig. 8—Mode-filter vane.

The $AR_{min}$ of an 8-inch section of 2.812 id nominally circular waveguide was measured for various values of $d_{max}-d_{min}$. The section was squeezed uniformly in such a way that the shape of the guide cross section was similar to an ellipse. The results obtained at 3,950 and 6,175 mc are shown in Figs. 3 and 4.

Using a waveguide section with a $AR_{min}$ of 25 db, the AR was measured for various orientations of the guide. The results are plotted in Fig. 5.

The $AR_T$ of two waveguide sections in series was also measured, both having an $AR_{min}$ of 25 db. One of the sections was held fixed in the orientation which gave $AR_{min}$, while various orientations were used for the other section. The values of $AR_T$ obtained are shown in Fig. 6.

It is seen that in most cases there is good agreement between calculated and measured values of AR. In Fig. 4 the measured AR is consistently higher than the calculated values. This is probably due to the difficulty in squeezing the guide uniformly over a length of exactly 8 inches.

The waveguide sections considered above were made short only for the sake of convenience in performing the measurements. Much longer sections could have been used, but to satisfy (12) a smaller value of $d_{max}-d_{min}$ would have been required.

In order to investigate the AR performance of longer waveguides, tests were made on fourteen 12.5-foot sections of 2.812 id nominally circular waveguide with $d_{max}-d'_{min} \leqq .004$ inch. The $AR_{min}$ of the individual sections were measured at 3,950 mc. The values obtained ranged from about 22 to 29 db.

The fourteen sections were connected to form a 175-foot run of waveguide. Each section was oriented so that its major axis was perpendicular to the major axis of the preceeding section to partially cancel the AR of the individual sections. The $AR_T$ was measured at 3,950 and 6,175 mc with the waveguide run rotated in steps through 180°. The results are plotted as $AR_T$ in db vs $\theta$ in degrees in Figs. 9 and 10.

The AR performance of the 175-foot run was improved by adding to it a short waveguide section. This section, referred to here as an AR compensator, was squeezed to have about the same $AR_{min}$ as the run itself

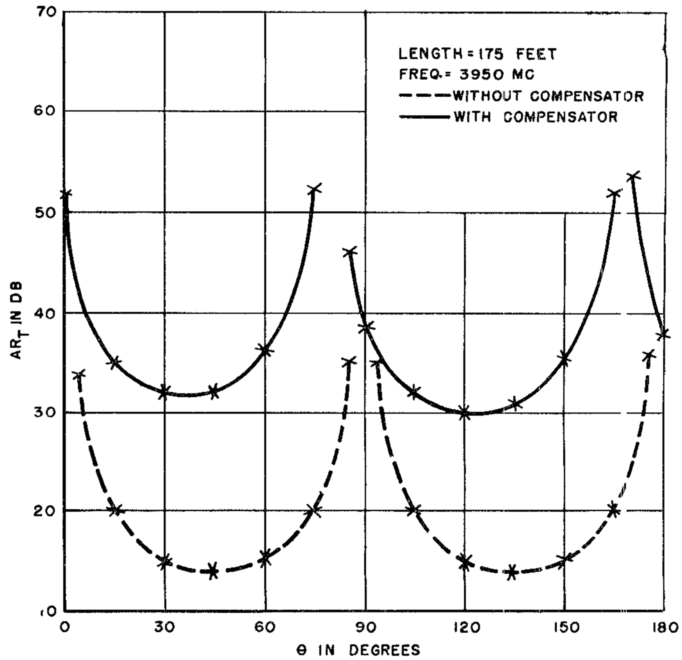


Fig. 9—Axial ratio at 3950 mc of a 175-foot run of 2.812 id nominally circular waveguide vs orientation of the run.

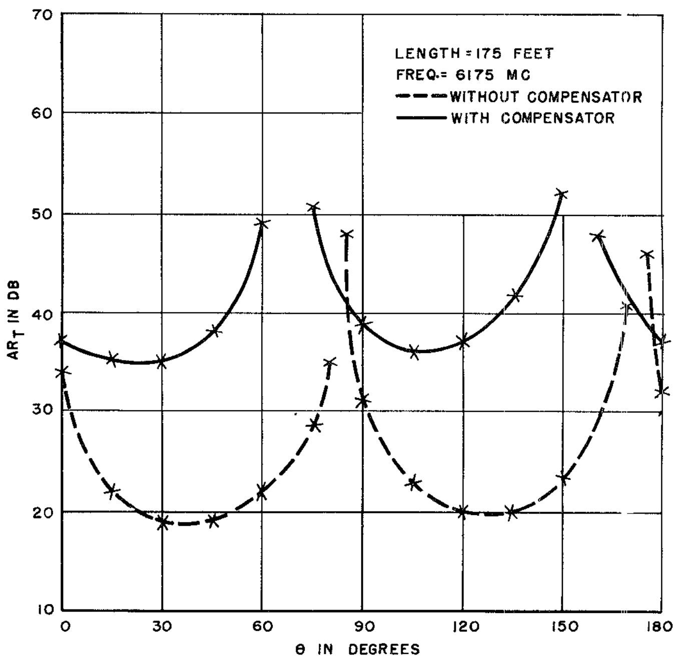


Fig. 10—Axial ratio at 6175 mc of a 175-foot run of 2.812 id nominally circular waveguide vs orientation of the run.

and oriented to increase the total $AR_{min}$ of the combination. With the compensator connected, the $AR_T$ was again measured at 3,950 and 6,175 mc for various orientations of the waveguide run. The values obtained are shown in Figs. 9 and 10.

The measurements show that the effect of ellipticity on the dominant-mode AR is a serious problem, even for very small values of $d_{max}-d_{min}$. The improvement resulting from the use of an AR compensator is clearly seen in Figs. 9 and 10. An even better performance could have been obtained by making the $AR_{min}$ of the compensator more nearly equal to the $AR_{min}$ of the waveguide run. The AR compensator is found to be an extremely broadbanded device.

### Conclusions

The dominant-mode AR in nominally circular waveguides can easily be obtained from approximate equations, when the difference between the major and minor axes is small and the waveguide is not too long. The calculated values of AR agree quite well with values found by measurements.

The effect of ellipticity on the dominant-mode AR, that is on the amount of crosstalk, is considerable. When waveguide sections are connected in series, the AR of the individual sections can be partially cancelled by properly orienting the sections relative to each other. The orientation of each waveguide section is independent of the frequency. The amount of cancellation, however, depends on what the values of minimum AR for the various sections are.

The AR performance of a waveguide run can be improved by using an AR compensator. If the minimum AR of the compensator is made about the same as the minimum AR of the run itself and is oriented properly, the total AR will be high for any polarization of the incident dominant-mode signals.

### Acknowledgment

The author wishes to thank many associates at the Bell Telephone Laboratories, in particular S. P. Morgan, Jr., for helpful criticism and suggestions in the preparation of this paper. The author is also indebted to those who assisted in making the measurements. Some tests were performed at Western Electric Company.

# The Design of Ridged Waveguides

SAMUEL HOPFER†

AS FAR as we are aware, the only published design information on ridged guide transmission lines is found in a paper by S. B. Cohn[1] and to some extent in the Waveguide Handbook.[2] Recent applications, however, have indicated a need for additional and, in some cases, more accurate design information. The present paper is largely written with this in mind.

The design curves presented here differ in several respects from those found in the literature. The more important differences can be stated as follows:

1. The step discontinuity susceptance is properly included in all calculations. Omission of this effect in calculating the cut-off frequencies of the higher modes, as well as in the calculation of the power carrying capacity, leads to considerable errors.
2. The attenuation calculations are based on a more rigorous expression for ridged guide attenuation.
3. The power handling curves take proper account of the breakdown at the edges.
4. The ridged guide impedance definition is different and seems more in line with experimental results.
5. The data are presented in terms of those parameters most likely to be specified in practice.

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[1] S. B. Cohn, "Properties of ridge waveguide," Proc. IRE, vol. 35, pp. 783–788; August, 1947.

[2] Nathan Marcuvitz, Waveguide Handbook, MIT Rad. Lab. Series, vol. 10, pp. 399–402.

### Cutoff Curves as a Function of Ridged Guide Geometry

Figs. 1(a) and 1(b) show the single- and double-ridged cross sections; their equivalent circuit representation is shown in Fig. 1(c). In keeping with common practice,

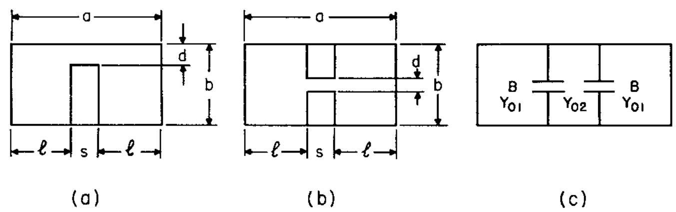


Fig. 1

the ridged guide modes are given the same designations as the corresponding modes in the rectangular waveguide. The equations which govern the cutoff conditions of the $TE_{no}$ type of modes are given by

$$\cot \kappa_x l - \frac{b}{d} \tan \kappa_x s/2 - B/Y_{01} = 0 \quad (1)$$

$$\cot \kappa_x l + \frac{b}{d} \cot \kappa_x s/2 - B/Y_{01} = 0. \quad (2)$$

Eq. (1) applies to the odd $TE_{no}$ modes and (2) applies to the even $TE_{no}$ modes. $\kappa_x$ is the propagation constant in the $x$ direction at cutoff and is given by $\kappa_x = 2\pi/\lambda_c$.